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RELIABILITY ANALYSIS OF RODDING ANODE PLANT
IN ALUMINUM INDUSTRY

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Abstract - The paper presents a reliability analysis of rodding anode plant in aluminum industry. Manufacturing process of raw aluminum passes through eight stations viz., butt shot blast station 1, butt & thimble removal press station 2 with standby arrangement, combined btp (butt & thimble press) station 3, stub straighten station 4, stub shot blast station 5, stub coating and drying station 6, casting station 7, and anode rod inspection station 8. Failure of any of the stations brings the system to a complete halt, except the butt & thimble removal press stations because of the parallel standby arrangement, and does not affect the system operation completely unless both fails. Six years maintenance data on component failures, repairs and associated costs are collected for the purpose of this analysis and various rates are estimated from the data. Measures of system effectiveness such as mean time to plant failure (MTPF), availability of the plant, busy period of repairman and expected number of repair have been obtained. Semi-Markov processes and regenerative point techniques are used in the analysis.

Keywords - reliability, semi-Markov process, regenerative point technique, failure, repair

NOTATIONS:

\( O_i \) Unit is operative at state \( i \)

\( S_2 \) One machine at station 2 in standby mode

\( 2r \) One machine at station 2 is under repair

\( D_i \) Down at state \( i \)

\( F_i, F_i^w \) Station \( i \) is failed and is under repair, waiting for repair.

\( f_i(t), F_i(t) \) p.d.f and c.d.f of failure rate of the station \( i \)

\( g_i(t), G_i(t) \) p.d.f and c.d.f of repair rate of the station \( i \)

\( q_i, Q_i \) Probability density function (p.d.f), cumulative distribution function (c.d.f) from a regenerative state \( i \) to a regenerative state \( j \) without visiting any other regenerative state \((0, t] \).

\( q_i^{(k)}, Q_i^{(k)} \) Probability density function (p.d.f), cumulative distribution function (c.d.f) from a regenerative state \( i \) to a regenerative state \( j \) with visiting \((k)\) state in \((0, t] \).

\( p_i, P_i^{(k)} \) Probability of transition from a regenerative state \( i \) to a regenerative state \( j \) without visiting any other state in \((0, t] \), probability of transition from a regenerative state \( i \) to a regenerative state \( j \) with visiting \((k)\) state in \((0, t] \).

\( \mathfrak{L} \) Symbol for Laplace Transform

\( \mathfrak{S} \) Symbol for Laplace-Stieltjes transform

\( m_i, m_i^{(k)} \) The unconditional mean time taken to transit to any regenerative state from the epoch of entry into regenerative state \( j \) without visiting any failed states, visiting failed state \( k \) once.

\( \mu_i \) Mean sojourn time in the regenerative state \( i \) before transiting to any other state.

\( \mathfrak{C} \) Laplace convolution.

\( \mathfrak{S} \) Stieltjes convolution.

\( \phi_i(t) \) Cumulative distribution function (c.d.f) of the first passage time from a regenerative state \( i \) to a failed state

\( M_i(t) \) The probability that the system initially up in regenerative state \( i \), is up at a time \( t \) without going to any regenerative state

\( A_i(t) \) The probability of the unit entering into upstate at instant \( t \), giving that the unit entered in regenerative state \( i \) at \( t = 0 \)

\( B_i(t) \) Probability that the repairman is busy in inspection of instant \( t \), given that the system entered regenerative state \( i \) at \( t = 0 \)

\( V_i(t) \) Expected number of visits of the repairman, given that the system entered regenerative state \( i \) at \( t = 0 \)

\( W_i(t) \) Probability that that the repairman is busy in regenerative state \( i \) at time \( t \) without passing any other regenerative state.

INTRODUCTION

Reliability analysis of industrial systems has been widely presented in the literature by many researchers due to its potential importance in industries. Complex systems are subject to failures because of many reasons which affect the profitability of the manufacturing industry and hence reliability analysis plays an important role in understanding the system performance while dealing with real industrial problems under different operating conditions and assumptions. Gulshan et al. [1] wrote about system analysis with perfect repair under partial failure mode and priority for repair to completely failed unit, Attahiru & Zhao [2] considered repairable system with three-units, Tutela et al. [3]-[5] worked for two-units system with regular repairman who is not always available, system with perfect repair at partial failure or complete failure mode, and the profit evaluation of a two-units cold standby system with tiredness and two types of repairmen. Rizwan et al. [6],[13] analyzed cold and hot standby systems with single-unit and two-units under different failure and repair situations where the reliability indices of interest are obtained and the cost benefit analysis of the systems are carried out. Mathew et al. [14]-[20] extensively analyzed the continuous casting plant and studied the variations under different operating conditions of the plant. Detailed analysis was reported for desalination plant by Padmavathi et al. [21] with online repair under emergency shutdowns, Rizwan et al. [22] with repair/maintenance strategy on first come first served basis, Padmavathi et al. [23]-[27] continued on desalination plant with priority for repair over maintenance, comparative analysis between the plant models, analysis under major and minor failures consideration, analysis by prioritizing repair over maintenance under major / minor failures, and comparative analysis between the plant models portraying two operating conditions of the plant as to which model is better than the other. The methodology was further extended for various industrial systems analyses by Gupta and Gupta [28] with post inspection concept, Ram et al. [29] waiting repair strategy, Malhotra and Taneja [30] both units operative on demand, Niwas et al. [31] obtained mean time to system failure and profit of a single unit...
system with inspection for feasibility of repair beyond warranty. Later, Rizwan et al. [32]-[34] focused on waste water treatment plant & anaerobic batch reactor reliability indices of interest were obtained in order to assess the plant/reactor performance. Recently, Taj et al. [35] analyzed a single machine subsystem of a cable plant with six maintenance categories. Thus, the methodology for industrial system analysis under various failure and maintenance situations has been widely presented in the literature and proved to be a good tool for industrial system/plant analysis under different operating conditions.

Aluminum being widely used as a source input for many manufacturing industries is a good reason for this analysis from reliability perspective. One such aluminum manufacturing industrial plant operating in Oman has been considered for this purpose, and analysis is carried out in order to understand the operating behavior of the plant. The anode rod plant manufactures raw aluminum blocks. Six years maintenance data on component failures, repairs and various associated costs are collected for the purpose of this analysis. Failure rates of the components, repairs and various associated costs are estimated from the data. Plant has eight stations viz., butt shot blast station 1, butt & thimble removal press station 2 with standby arrangement, combined bpt (butt & thimble press) station 3, stub straighten station 4, stub shot blast station 5, stub coating and drying station 6, casting station 7, and anode rod inspection station 8. The plant operates round the clock, and failure in any of the stations impacts the plant to a complete shutdown situation. However, butt & thimble removal press station 2, has standby arrangement, and do not affect the system operation completely unless both of them fail. The state transition diagram of the plant is shown in Fig. 1. Based on the various operating states of the plant, a detailed analysis is carried out using semi Markov process and regeneration point techniques. Outcome of the entire analysis is measured in terms of overall system effectiveness such as mean time to plant failure (MTPF), availability of the plant, expected busy period of the repairman, and expected number of repairs.

The data summary reflects the following related estimations:

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Station No.</th>
<th>Repair rate</th>
<th>Failure rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Station 1</td>
<td>a₁ = 0.11382393</td>
<td>λ₁ = 0.01768162</td>
</tr>
<tr>
<td>2</td>
<td>Station 2</td>
<td>a₂ = 0.18064556</td>
<td>λ₂ = 0.02045033</td>
</tr>
<tr>
<td>3</td>
<td>Station 3</td>
<td>a₃ = 0.21289838</td>
<td>λ₃ = 0.00057936</td>
</tr>
<tr>
<td>4</td>
<td>Station 4</td>
<td>a₄ = 0.21138211</td>
<td>λ₄ = 0.01449055</td>
</tr>
<tr>
<td>5</td>
<td>Station 5</td>
<td>a₅ = 0.18921776</td>
<td>λ₅ = 0.00327232</td>
</tr>
<tr>
<td>6</td>
<td>Station 6</td>
<td>a₆ = 0.231620</td>
<td>λ₆ = 0.003358</td>
</tr>
<tr>
<td>7</td>
<td>Station 7</td>
<td>a₇ = 0.24061465</td>
<td>λ₇ = 0.000341</td>
</tr>
<tr>
<td>8</td>
<td>Station 8</td>
<td>a₈ = 0.11382393</td>
<td>λ₈ = 0.01768162</td>
</tr>
</tbody>
</table>

MODEL DESCRIPTIONS AND ASSUMPTIONS

1) Initially at state 0, the system is in good and operational mode.
2) Only station 2 has the standby arrangement.
3) Failure at any station other than station 2, leads to a complete shutdown state.
4) All necessary maintenances are off-line which means necessary repairs or replacements need plant, to be in switch-off mode.
5) Maintenances are all random and need to be addressed on requirement by a single repairman.
6) The failure of any component at a particular station is taken as a single failure event.
7) All failure times are assumed to have exponential distribution whereas other times are general.
8) After each repair the system works as good as new and returns to new state.
9) Repairman comes as soon as a unit or component fails, and other failures need to wait until previous failures have been resolved.

TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Possible transitions states of the plant are shown in Fig. 1. States 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 and 16 are the failed states whereas 0 & 2 are the operating states. The epochs of entry into states 0, 1, 2, 3, 4, 5, 6, 7 and 8 are regeneration points and thus are regenerative states.

The transition probabilities are given by:

$$Q(t) = \lambda t e^{-\lambda t} \frac{\lambda t}{1!}$$

$$(1)$$

$$Q(t) = \lambda t e^{-\lambda t} \frac{\lambda t}{1!}$$

$$(2)$$

$$Q(t) = \lambda t e^{-\lambda t} \frac{\lambda t}{1!}$$

$$(3)$$

$$Q(t) = \lambda t e^{-\lambda t} \frac{\lambda t}{1!}$$

$$(4)$$

$$Q(t) = \lambda t e^{-\lambda t} \frac{\lambda t}{1!}$$

$$(5)$$

$$Q(t) = \lambda t e^{-\lambda t} \frac{\lambda t}{1!}$$

$$(6)$$

$$Q(t) = \lambda t e^{-\lambda t} \frac{\lambda t}{1!}$$

$$(7)$$

$$Q(t) = \lambda t e^{-\lambda t} \frac{\lambda t}{1!}$$

$$(8)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(9)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(10)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(11)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(12)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(13)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(14)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(15)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(16)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(17)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(18)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(19)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(20)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(21)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(22)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(23)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(24)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(25)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(26)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(27)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(28)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(29)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(30)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(31)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(32)$$

$$Q(t) = g_s(t)\text{d}t$$

$$(33)$$

The non-zero element $p_{11}$ can be obtained by:

$$p_{11} = \lim_{s \to 0} \int_{0}^{s} q_s \text{d}t \text{ or } p_{11} = \lim_{s \to 0} q_s$$

$$(34)$$

$$p_{12} + p_{12} + p_{12} + p_{12} + p_{12} + p_{12} + p_{12} + p_{12} = 1$$

$$(35)$$

$$p_{11} + p_{12} + p_{12} + p_{12} + p_{12} + p_{12} + p_{12} + p_{12} = 1$$

$$(36)$$

$$p_{11} + p_{12} + p_{12} + p_{12} + p_{12} + p_{12} + p_{12} + p_{12} = 1$$

$$(37)$$

The mean sojourn time is the expected time taken by the system in a particular state before transition to any other state and it is also called as the mean survival time in the state,

$$\mu = E(t) = P(T > t) = \int_{t}^{\infty} q(t) \text{d}t \text{ fin state } i \text{ is given by:}$$

$$\mu = \frac{1}{\lambda t}$$

$$(38)$$

$$\mu = \frac{1}{\lambda t}$$

$$(39)$$

$$\mu = \frac{1}{\lambda t}$$

$$(40)$$

$$\mu = \frac{1}{\lambda t}$$

$$(41)$$

$$\mu = \frac{1}{\lambda t}$$

$$(42)$$

$$\mu = \frac{1}{\lambda t}$$

$$(43)$$

$$\mu = \frac{1}{\lambda t}$$

$$(44)$$
\[ \mu_i = \frac{1}{n_i} \quad (45) \]
\[ \mu_0 = \frac{1}{n_0} \quad (46) \]

The unconditional mean time \( m_j = \sum_{i=0}^\infty t \cdot \frac{d}{dt} q_i(j) \) taken by the system to transit for any state \( j \) when it has taken from epoch of entrance into regenerative state \( i \) is mathematically stated as:

\[ m_{0 \to 1} + m_{0 \to 2} + \cdots + m_{0 \to n} = \mu_0 \quad (47) \]
\[ m_{1 \to 1} = \mu_1 \quad (48) \]
\[ m_{2 \to 1} = \mu_2 \quad (49) \]
\[ m_{3 \to 1} = \mu_3 \quad (50) \]
\[ m_{4 \to 1} = \mu_4 \quad (51) \]
\[ m_{5 \to 1} = \mu_5 \quad (52) \]
\[ m_{6 \to 1} = \mu_6 \quad (53) \]
\[ m_{7 \to 1} = \mu_7 \quad (54) \]
\[ m_{8 \to 1} + m_{8 \to 2} + m_{8 \to 3} + m_{8 \to 4} + m_{8 \to 5} + m_{8 \to 6} + m_{8 \to 7} = \mu_8 \quad (55) \]

**THE MATHEMATICAL ANALYSIS**

1. **MEAN TIME TO PLANT FAILURE**

Regarding the failed states as absorbing states and employing the arguments used for probabilistic processes, we have the following recursive relations for \( \Phi_0 \). Cumulative time from \( i \) to a failed state

\[ \Phi_0(t) = \sum_{j=1}^{n_i} Q_{0j} + Q_{00} \Phi_0(t) \quad (56) \]
\[ \Phi_2(t) = Q_{20} \Phi_0(t) + \sum_{j \neq 0} Q_{2j} \quad (57) \]

Taking Laplace-Stieltjes Transform (L.S.T) after the above equations 56 & 57 and solving for \( \Phi_0^{-1} \), the mean time to system failure when the system starts from the state 0 is given by:

\[ MTS = \lim_{s \to 0} \frac{s \cdot \Phi_0^{-1}(s)}{s} \quad (58) \]
\[ MTS = \lim_{s \to 0} \frac{s \cdot \Phi_2^{-1}(s)}{s} = \lim_{s \to 0} \frac{s \cdot \Phi_0^{-1}(s)}{s \cdot \Phi_0^{-1}(s)} = \frac{N}{D} = 1 - p_0 \quad (59) \]

Taking Laplace-Stieltjes Transform (L.S.T) of the above equations and solving for \( \Phi_0^{-1} \), the mean time to system failure when the system starts from the state 0 is given by:

\[ M = \frac{N}{D} = 1 - p_0 \quad (60) \]

2. **AVAILABILITY ANALYSIS OF THE PLANT**

Using the probabilistic arguments and defining \( A_i(t) \) as the probability of the unit entering into upstate at instant \( t \), giving that the unit entered in regenerative state \( i \) at \( t = 0 \), the following recursive relations are obtained:

\[ A_0(t) = M_0(t) + q_0 \cdot \Phi_0(t) \cdot A_0(t) + q_0 \cdot \Phi_0(t) \cdot A_0(t) + q_0 \cdot \Phi_0(t) \cdot A_0(t) + q_0 \cdot \Phi_0(t) \cdot A_0(t) + q_0 \cdot \Phi_0(t) \cdot A_0(t) + q_0 \cdot \Phi_0(t) \cdot A_0(t) + q_0 \cdot \Phi_0(t) \cdot A_0(t) + q_0 \cdot \Phi_0(t) \cdot A_0(t) \quad (61) \]

**Here,**

\[ M_0(t) = e^{-\lambda_0 t} + \lambda_0 t \quad (62) \]
\[ M_0(t) = e^{-\lambda_0 t} + \lambda_0 t \quad (63) \]
\[ M_0(t) = e^{-\lambda_0 t} + \lambda_0 t \quad (64) \]
\[ M_0(t) = e^{-\lambda_0 t} + \lambda_0 t \quad (65) \]
\[ M_0(t) = e^{-\lambda_0 t} + \lambda_0 t \quad (66) \]
\[ M_0(t) = e^{-\lambda_0 t} + \lambda_0 t \quad (67) \]
\[ M_0(t) = e^{-\lambda_0 t} + \lambda_0 t \quad (68) \]

3. **BUSY PERIOD ANALYSIS OF REPAIRMAN**

Using the probabilistic arguments and defining \( B_i(t) \) as the probability that the repairman is busy for repair at instant \( t \), given that the unit entered in regenerative state \( i \) at \( t = 0 \), the following recursive relations are obtained:

\[ B_0(t) = q_0 \cdot \Phi_0(t) + q_0 \cdot \Phi_0(t) + q_0 \cdot \Phi_0(t) + q_0 \cdot \Phi_0(t) + q_0 \cdot \Phi_0(t) + q_0 \cdot \Phi_0(t) + q_0 \cdot \Phi_0(t) + q_0 \cdot \Phi_0(t) + q_0 \cdot \Phi_0(t) \quad (69) \]

**Here,**

\[ B_0(t) = \Phi_0(t) \quad (70) \]
\[ B_0(t) = \Phi_0(t) \quad (71) \]
\[ B_0(t) = \Phi_0(t) \quad (72) \]
\[ B_0(t) = \Phi_0(t) \quad (73) \]
\[ B_0(t) = \Phi_0(t) \quad (74) \]
\[ B_0(t) = \Phi_0(t) \quad (75) \]
\[ B_0(t) = \Phi_0(t) \quad (76) \]
\[ B_0(t) = \Phi_0(t) \quad (77) \]

Taking Laplace-Stieltjes Transform (L.S.T) of the above equations and solving for \( B_i(t) \) then the busy period of the repairman is as:

\[ B_i(t) = q_0 \cdot \Phi_0(t) \quad (78) \]

**Here,**

\[ B_i(t) = \Phi_0(t) \quad (79) \]

**Where,**

\[ N_i(t) = p_0 \cdot \Phi_0(t) \quad (10) \]
\[ N_i(t) = p_0 \cdot \Phi_0(t) \quad (11) \]
\[ N_i(t) = p_0 \cdot \Phi_0(t) \quad (12) \]
\[ N_i(t) = p_0 \cdot \Phi_0(t) \quad (13) \]
\[ N_i(t) = p_0 \cdot \Phi_0(t) \quad (14) \]
\[ N_i(t) = p_0 \cdot \Phi_0(t) \quad (15) \]
\[ N_i(t) = p_0 \cdot \Phi_0(t) \quad (16) \]

and \( D_i(t) \) (Already specified)
4. EXPECTED NUMBER OF VISITS FOR REPAIR

Let $V_r(t)$ be defined as the expected number of visits for repairs in $(0, t)$, given that the system initially starts from the regenerative state $i$. Using the probabilistic arguments, the following recursive relations are obtained for $V_r(t)$:

$$V_1(t) = Q_0 I(t) + V_2(t)$$
$$V_2(t) = Q_0 S(t) + V_3(t) + Q_0 G(t) + V_4(t) + Q_0 D(t) + V_5(t) + Q_0 F(t) + V_6(t) + Q_0 H(t) + V_7(t)$$
$$\vdots$$

$$V_n(t) = Q_0 I(t) + V_{n+1}(t)$$

Taking Laplace-Stieltjes transform of the equations and solving for $V_r(s)$, then the busy period of the repairman is as: $\lim_{s \to 0^+} s^{-1} V_r(s)$,

$$V_0 = \lim_{s \to 0^+} s^{-1} V_r(s) = \frac{N_0}{D_0} = \frac{N_2}{D_1}$$

Where,

$N_2 = (1 - p_1 - p_2 - p_3) + p_1 d_1 - p_2 d_2$

$D_1$ (Already specified)

PARTICULAR CASE

For graphical interpretation, the following particular cases are considered:

$g_1(t) = e^{-at}$, $g_2(t) = e^{-at}$, $g_3(t) = e^{-at}$, $g_4(t) = e^{-at}$, $g_5(t) = e^{-at}$, $g_6(t) = e^{-at}$

Using the data as summarized, the following values of measures of plant are obtained:

- Mean time to plant failure = 29.2721 hrs.
- Availability = 0.714265 hrs.
- Busy period of repairman = 0.19397
- Expected number of repairs = 0.0395482

CONCLUSIONS

Estimated reliability results facilitate the plant engineers in understanding the system behavior and thereby build up a scope for improvement in terms of plant performance by reviewing maintenance strategies. Mean time to plant is about 29 hours which shows, there is a failure almost every 29 hours and the company should really look into the reasons for such frequent failures and probably review the existing maintenance strategies. Other measures could further be looked into for better results from optimization perspective; but are somewhat within the tolerable limits. As a future direction, the analysis could further be explored for real complex situations as multiple components failure with a single repairman.
Figure 1
State Transition Diagram